Game Theory: Mixed strategies explained

What is a mixed strategy?

A mixed strategy exists in a strategic game, when the player does not choose one definite action, but rather, chooses according to a probability distribution over a his actions.

Imagine you are in Nandos, and you are considering of choosing Lemon & Herb or Wild Herb sauce for you chicken. The spiciness of the sauce is equal, but you are indifferent between the two flavours and thus you decide to toss a coin. The probability of you choosing Lemon & Herb is 50%, or 1/2. The probability of choosing Wild Herb is also 50%, or 1/2.

Note: In pure strategies, the player assigns 100% probability to one plan of action.

Why do we bother about mixed strategies?

There are cases where a pure strategy equilibrium does not exist. Therefore, we have to find the probability for which the player would be willing to randomise between his actions. This probability will depend on the expected payoffs of the player for each of his actions.

Finding the Mixed Strategy Nash Equilibrium: An example

Harry, Ron and Hermione are choosing courses for their new semester at Hogwarts. Harry and Ron are interested in the course "Badass Fighting Poses". On the other hand, Hermione is interested in the course "Introduction to Magic Economics". However, they prefer taking the same course than taking different courses, as they will be together (Harry and Ron will freeload on Hermione's perfect notes, while Hermione wants to constantly scold them on taking her perfect notes). Their respective payoffs are illustrated in the matrix below:

How to read: *(4,3) means that Harry and Ron receive a payoff of 4, and Hermione a payoff of 3 when they both pick Badass Fighting Poses.*

		Hermione's Choice	
		Badass Fighting Poses	Magic Economics
Harry and Ron's Choice	Badass Fighting Poses	(4, 3)	(1,1)
	Magic Economics	(0,0)	(3,4)

This game has two Nash Equilibria, when all choose Badass Fighting Poses, and when all choose Magic Economics. Thus, we have a Coordination Problem. When will the three students choose Badass Fighting Poses and when will they choose Magic Economics?

In order to find the Mixed Strategy Equilibrium, we first have to find the probability that each of the players assigns to each action. This will be done by calculating the Expected Payoffs of Harry and Ron, and Hermione respectively.

For Harry and Ron we assume that they choose Badass Fighting Poses with probability x. This implies that the probability with which they choose Magic Economics is (1 - x). Similarly, we assume that Hermione chooses Badass Fighting Poses with probability y and Magic Economics with probability (1 - y). We illustrate this at the graph below:

		y	1-y
		Badass Fighting Poses	Magic Economics
x	Badass Fighting Poses	(4, 3)	(1,1)
1-x	Magic Economics	(0,0)	(3,4)

Calculate the expected payoffs

For **Harry and Ron**, the Expected Payoff of this game is the sum of the payoffs of the two possible actions, multiplied with the probability of Hermione choosing those actions:

EP{Badass Fighting Poses} = $4y + 1(1 - y) = 3y + 1$

EP{Magic Economics} = $0y + 3(1 - y) = 3 - 3y$

For Harry and Ron to choose Badass Fighting Poses, the expected payoff of this course must be higher than the expected payoff for Magic Economics:

$3y + 1 > 3 - 3y$

$6y > 2$

$y > 1/3$

We do the similar thing for **Hermione**, but this time we multiply her payoffs with the probabilities of Harry and Ron:

EP{Badass Fighting Poses} = $3x + 0(1 - x) = 3x$

$EP\{\text{Magic Economics}\} = 1x + 4(1 - x) = 4 - 3x$

Again, for Hermione to choose the Pure Nash Equilibrium of Badass Fighting Poses, it must be:

$3x > 4 - 3x$

$6x > 4$

$x > 2/3$

For y = 1/3 and x = 2/3, the three magicians are indifferent between the two options. Therefore, those probabilities are a Mixed Strategy Nash Equilibrium.

Beyond this example

- When you are asked to find the Nash Equilibria of a game, you first state the Pure Strategy Nash Equilibria, and then look for the mixed strategy one as well.
- Find the probabilities of the expected payoffs for each player with the method described above.
- If a player has three or more action plans, the probabilities you assign are x1, x2, x3,… where x1 + x2 + x3 + … = 1.